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FAILURE SURFACES IN INFINITE SLOPES

**Michael J. Gonsior, William S. Hartsog,
and Glen L. Martin**



USDA Forest Service Research Paper INT-150, 1974
INTERMOUNTAIN FOREST & RANGE
EXPERIMENT STATION
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ABSTRACT

General solutions for the failure surfaces in slopes where the assumptions of the infinite slope theory are valid have been derived. The solutions are applicable to layered systems and to any seepage conditions provided that both layering and ground water flow are parallel to ground slope. It is concluded that the infinite slope theory is of limited applicability unless modified. A modification to enable transition from the active to passive stress states is suggested.

INTRODUCTION AND PURPOSE

The infinite slope theory is based upon the assumption that a conjugate relationship exists between the vertical pressures and the lateral pressures acting on vertical planes, and that the stress conditions at any two points of equal depth are identical.

"Thus the stresses at various depths on any vertical plane must be the same as those at corresponding depths on any other vertical plane."
(Taylor 1948)

The infinite slope theory is often used to assess the stability of long, natural slopes wherein the soil properties are constant at a given depth below the surface of the slope. It appears to be most useful for situations wherein the soils are relatively uniform in depth and underlain by a stronger medium, such as bedrock.

Because no slopes are infinitely long, the applicability of this theory is a matter of judgment. It is recognized that, if failure is incipient, the lateral pressure has an active, or minimum, value at the upper end of the failing mass and a passive, or maximum, value at the lower end. In practice, however, the difference in end conditions is usually ignored, or else a simple correction factor is introduced to account for the difference.

Usually the criterion used to aid in judging applicability of the infinite slope theory is the ratio of length of slope to critical depth. A ratio of 20:1 commonly is used.

The purpose of this paper is to describe the failure surfaces in slopes that comply with the infinite slope theory and thereby, to aid the judgment of the user of this theory in assessing its applicability.

PREVIOUS WORK

The differential equation for the surface of rupture in an infinite cohesive earth slope was derived by Résal (1910) in 1910. His derivation was based upon a consideration of stress equilibrium on a triangular-shaped differential element within the slope (fig. 1), the sides of which are: (1) vertical, (2) parallel to the ground slope, and (3) parallel to the tangent of the rupture surface. In compliance with the infinite slope theory, the stress, r , acting on the vertical side of the element and the stress, p , acting on the side parallel to ground slope are conjugate. Thus, Résal's derivation enables determination of the normal (\bar{n}) and tangential (t) stresses acting on the assumed plane of rupture in terms of the conjugate vertical and lateral stresses. To assure compliance with the Coulomb assumption, it can be written that

$$t - \bar{n} \tan \phi = c \dots\dots\dots (1)$$

where ϕ is the angle of internal friction and c is the cohesive strength. To determine the angle of the tangent to the rupture surface, it is a simple matter to maximize the function, $t - \bar{n} \tan \phi$, by differentiating the Coulomb expression; thus

$$\frac{dt}{d\alpha'} - \frac{d\bar{n}}{d\alpha'} \tan \phi = 0 \dots\dots\dots (2)$$

where α' is the angle between the ground surface and the tangent to the rupture surface, and is related to the failure surface by the expression

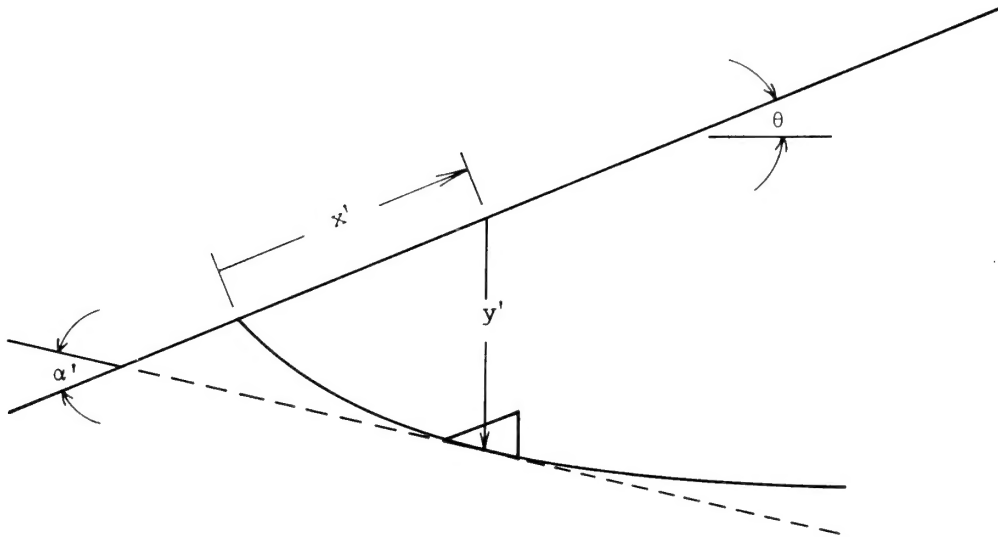
$$dx' = \frac{\cos (\theta - \alpha')}{\sin \alpha'} dy' \dots\dots\dots (3)$$

where x' is the distance from the intersection of the rupture surface with the ground surface to the point of consideration measured parallel to the ground surface, y' is the vertical distance from ground surface to the point of consideration, and θ is the slope of the ground surface.

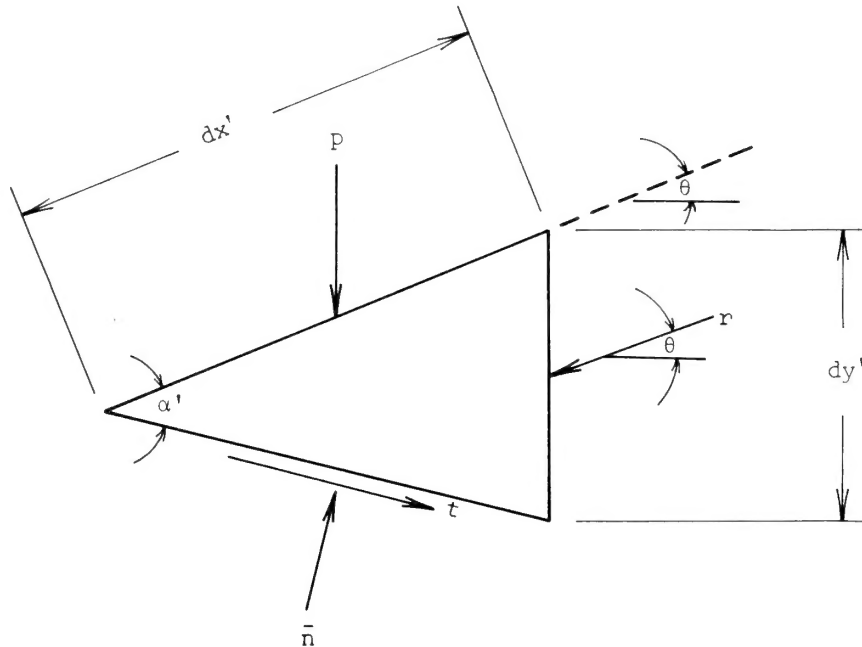
By substituting the values of t and \bar{n} (determined from the stress equilibrium considerations and expressed in terms of p and r) into the above equations, Résal was led to the differential equation

$$\begin{aligned} & - \cos (\alpha' - \theta) \sin (\alpha' - \theta + \phi) \cos (2\alpha' + \phi) \\ & + \sin \alpha' \cos (\alpha' + \phi) \cos (2\alpha' - 2\theta + \phi) \\ & = \frac{c}{\gamma y} \cos \phi \cos (2\alpha' + \phi) \dots\dots\dots (4) \end{aligned}$$

where γ is the unit weight of the soil. Résal concluded that this equation was not integrable.



(a) Location of element on hypothetical failure surface.



(b) Definition of element and stresses.

Figure 1.--Element definition for Résal's derivation.

In 1922, Frontard (1922) succeeded in solving equation 4 for the coordinates of the passive failure surface under the conditions of the Rankine-Résal stress assumptions. His solution was presented as follows:

$$x' = \frac{c \cos \phi \tan \theta (\lambda_0 \cos \theta + \cos \phi \sin \lambda_0 - \lambda \cos \theta - \cos \phi \sin \lambda)}{\gamma \sin (\theta - \phi) \sqrt{\sin (\theta - \phi) \sin (\theta + \phi)}} \dots \dots \dots (5a)$$

$$y' = \frac{c \cos \phi \tan \theta}{\gamma \sin (\theta - \phi) \sin (\theta + \phi)} \left[\frac{\sin \phi}{\tan \theta} - \sqrt{\sin (\theta - \phi) \sin (\theta + \phi)} \sin \lambda + \cos \theta \cos \lambda \right] \dots \dots \dots (5b)$$

where x' and y' are the coordinates in the directions parallel to the ground slope and vertically below ground slope, respectively.

The parameter λ is defined by the expression

$$\tan \frac{\lambda}{2} = \sqrt{\frac{\sin \theta + \sin \phi}{\sin \theta - \sin \phi}} \tan \left(\alpha' - \frac{\theta - \phi}{2} \right) \dots \dots \dots (6a)$$

Corresponding to the coordinates $x' = y' = 0$, the angle $\alpha' = \pi/4 - \phi/2$ in the passive case and accordingly, the parameter λ_0 is defined by the expression

$$\tan \frac{\lambda_0}{2} = \sqrt{\frac{\sin \theta + \sin \phi}{\sin \theta - \sin \phi}} \tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right) \dots \dots \dots (6b)$$

By using Frontard's approach, similar expressions can be derived for the active failure surface. They are as follows:

$$x' = \frac{c \cos \phi \tan \theta (\lambda_0 \cos \theta + \cos \phi \sin \lambda_0 - \lambda \cos \theta - \cos \phi \sin \lambda)}{\gamma \sin (\theta - \phi) \sqrt{\sin (\theta - \phi) \sin (\theta + \phi)}} \dots \dots \dots (7a)$$

$$y' = \frac{c \cos \phi \tan \theta}{\gamma \sin (\theta - \phi) \sin (\theta + \phi)} \left[\frac{\sin \phi}{\tan \theta} + \sqrt{\sin (\theta - \phi) \sin (\theta + \phi)} \sin \lambda + \cos \theta \cos \lambda \right] \dots \dots \dots (7b)$$

$$\tan \frac{\lambda}{2} = \sqrt{\frac{\sin \theta + \sin \phi}{\sin \theta - \sin \phi}} \tan \left(\alpha' + \frac{\theta - \phi}{2} \right) \dots \dots \dots (8a)$$

$$\tan \frac{\lambda_0}{2} = \sqrt{\frac{\sin \theta + \sin \phi}{\sin \theta - \sin \phi}} \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right) \dots \dots \dots (8b)$$

In 1936, Terzaghi (1936) presented a discussion of Frontard's solution in connection with the analysis of simple slopes. He implied that Frontard had combined the vertical projections of the active and passive failure surfaces to obtain the critical height of simple slopes. However, Frontard made no statements in his 1922 paper with regard to the application of his work to simple slopes; nor, indeed, did he mention the active failure surface or present the solution for same.

Terzaghi correctly criticized attempts to combine the active and passive curves obtained by this procedure for, by the assumptions of the infinite slope theory, the active and passive states of stress cannot exist simultaneously in a slope. However, he was incorrect in attributing any statements to Frontard regarding the applicability of this theory to real slopes.

Hartsog and Martin (1974) presented general solutions for the active and passive earth pressures in an infinite slope for cases with and without seepage forces. They used the Mohr stress coordinates as a basis for their work, and derived expressions for the state of stress on a plane oriented at any angle under conditions of either active or passive failure. Their paper formed the basis for the following development.

The active and passive stress circles for the general case of a point located beneath the phreatic surface when ground water flows parallel to the ground slope are shown in figure 2. Note that $\bar{\sigma}_a$ and τ_a are the effective normal and shear stresses, respectively, acting on a plane parallel to ground surface at the point or depth being considered. For a depth less than that to ground water, the coordinates $(\bar{\sigma}_a, \tau_a)$ would simply lie along the line extending from the origin of stresses at an angle, θ , to the $\bar{\sigma}$, or normal stress, axis.

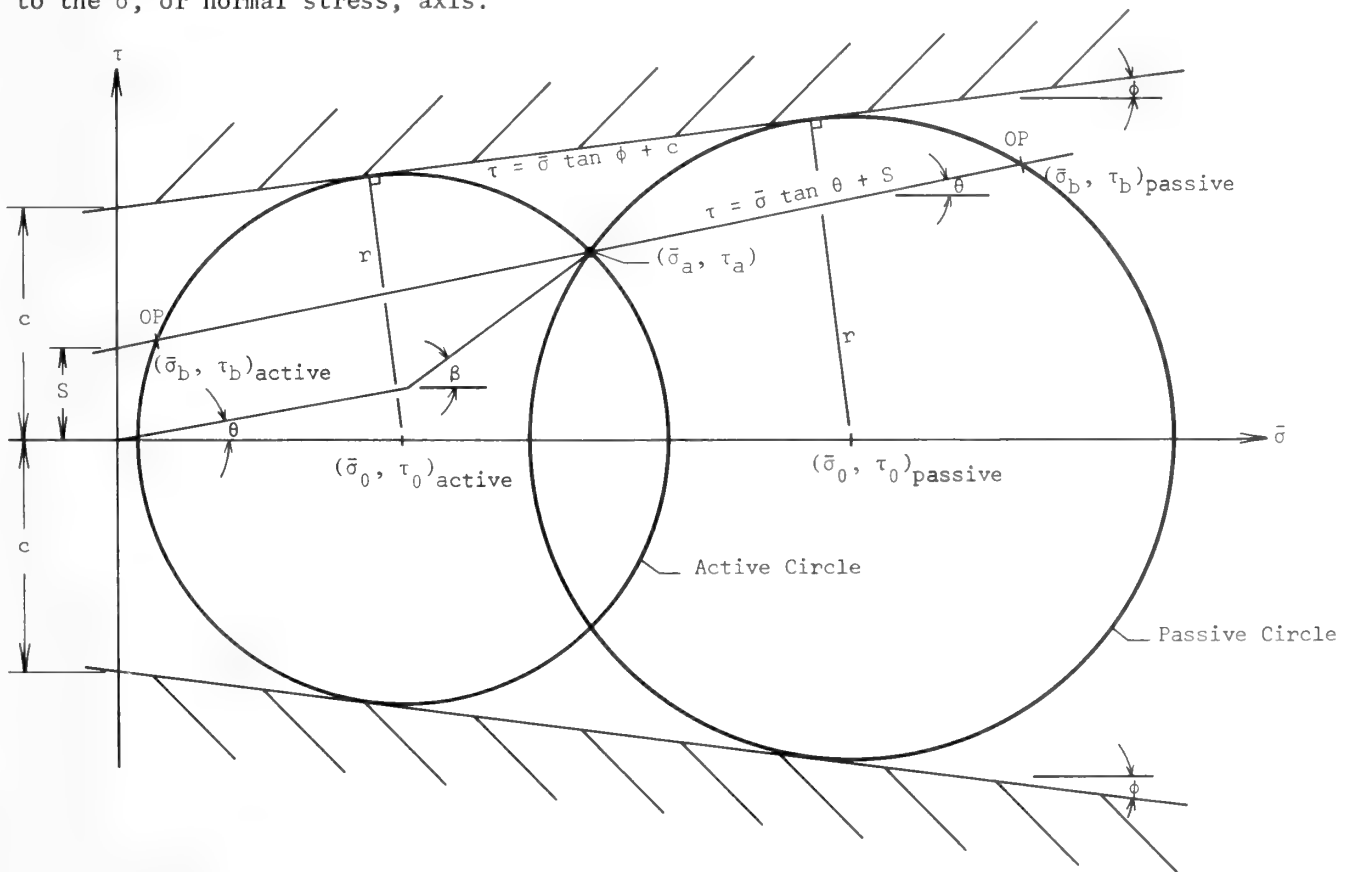


Figure 2.--Active and passive stress circles and their relationship to Mohr's rupture envelope. {After Hartsog and Martin (2)}.

The angle, β , shown in figure 2, can be expressed as

$$\beta = \tan^{-1} \left[\left(1 + \frac{\gamma_w}{\gamma_b} \right) \tan \theta \right] \dots \dots \dots (9)$$

and the seepage force per unit area, S , is given by

$$S = \gamma_w \sin \theta (z - z_w) \cos \theta \dots \dots \dots (10)$$

where z is the vertical depth below ground surface to the point being considered, z_w is the vertical depth below ground surface to the phreatic surface, and γ_w is the unit weight of water.

By defining the angle, α , as shown in figure 3, Hartsog and Martin derived the following general expressions for the stresses $\bar{\sigma}_c$ and τ_c on a plane oriented at any angle, α , at depth z :

$$\bar{\sigma}_c = \frac{2 \bar{\sigma}_0 + \bar{\sigma}_b (-1 + \tan^2 \alpha - 2 \tan \alpha \tan \theta) - 2 S \tan \alpha}{1 + \tan^2 \alpha} \dots \dots \dots (11)$$

$$\tau_c = \tau_b + (\bar{\sigma}_c - \bar{\sigma}_b) \tan \alpha \dots \dots \dots (12)$$

where

$$\bar{\sigma}_b = \frac{2 \bar{\sigma}_0 - 2 S \tan \theta}{(1 + \tan^2 \theta)} - \bar{\sigma}_a \dots \dots \dots (13)$$

$$\tau_b = \bar{\sigma}_b \tan \theta + S \dots \dots \dots (14)$$

$$\bar{\sigma}_a = \gamma_t z_w \cos^2 \theta + \gamma_b (z - z_w) \cos^2 \theta \dots \dots \dots (15)$$

$$\begin{aligned} \bar{\sigma}_0 = & \bar{\sigma}_a (1 + \tan^2 \phi) + c \tan \phi \\ & \pm \{ [- \bar{\sigma}_a (1 + \tan^2 \phi) - c \tan \phi]^2 \\ & - [\bar{\sigma}_a^2 (1 + \tan^2 \phi + \tan^2 \theta + \tan^2 \phi \tan^2 \theta) \\ & + 2 \bar{\sigma}_a S \tan \theta (1 + \tan^2 \phi) \\ & + S^2 (1 + \tan^2 \phi) - c^2] \}^{\frac{1}{2}} \dots \dots \dots (16) \end{aligned}$$

In the above expressions, γ_t is the unit weight of the soil above ground water level and γ_b is the buoyant unit weight of the soil below ground water level. The (+) and (-) signs in equation 16 correspond to the passive and active stress circles, respectively.

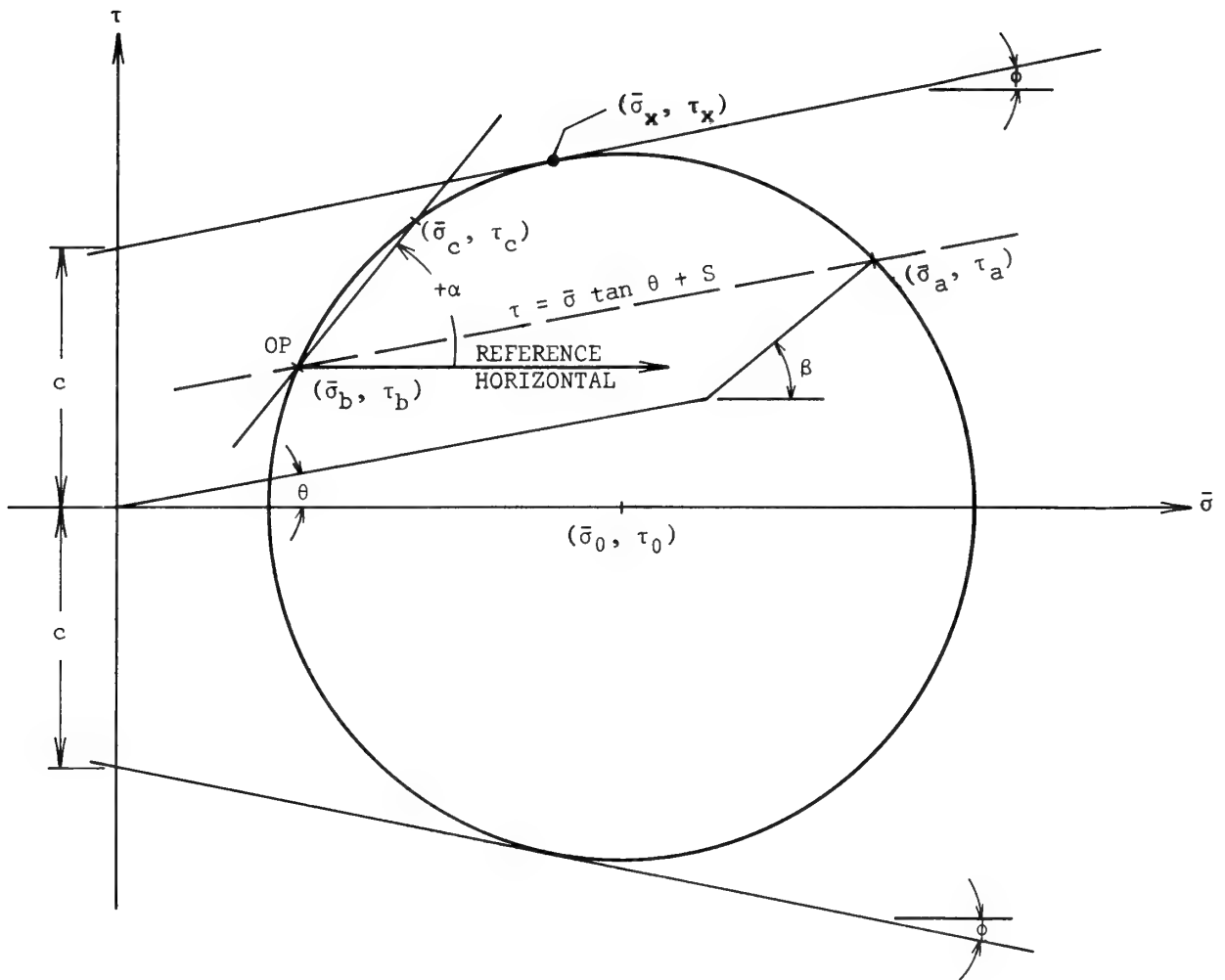


Figure 3.--Typical stress circle. {After Hartsog and Martin (2)}.

GENERAL SOLUTION

In figure 3, the point of tangency of the stress circle with the strength envelope is defined by the coordinates $(\bar{\sigma}_x, \tau_x)$. The tangent of the angle, α_f , of the line connecting the origin of planes (OP) and the point of tangency can be expressed as

$$m = \tan \alpha_f = \frac{\tau_x - \tau_b}{\bar{\sigma}_x - \bar{\sigma}_b} \dots \dots \dots (17)$$

It can be shown that

$$\bar{\sigma}_x = \frac{\bar{\sigma}_0 - c \tan \phi}{1 + \tan^2 \phi} \dots \dots \dots (18)$$

and

$$\tau_x = \left(\frac{\bar{\sigma}_0 - c \tan \phi}{1 + \tan^2 \phi} \right) \tan \phi + c \dots \dots \dots (19)$$

Combining equations 13, 14, 18, and 19, the expression for m can be written:

$$\begin{aligned} & \bar{\sigma}_0 [\tan \phi (1 + \tan^2 \theta) - 2 \tan \theta (1 + \tan^2 \phi)] \\ & + \bar{\sigma}_a [\tan \theta (1 + \tan^2 \phi) (1 + \tan^2 \theta)] \\ & + c [(1 + \tan^2 \phi) (1 + \tan^2 \theta) \\ & \quad - \tan^2 \phi (1 + \tan^2 \theta)] \\ & + S [2 \tan^2 \theta (1 + \tan^2 \phi) \\ & \quad - (1 + \tan^2 \phi) (1 + \tan^2 \theta)] \\ m = & \frac{\bar{\sigma}_0 [(1 + \tan^2 \theta) - 2 (1 + \tan^2 \phi)]}{\dots \dots \dots} \\ & + \bar{\sigma}_a [(1 + \tan^2 \phi) (1 + \tan^2 \theta)] \\ & - c [\tan \phi (1 + \tan^2 \theta)] \\ & + S [2 \tan \theta (1 + \tan^2 \phi)] \dots \dots \dots (20) \end{aligned}$$

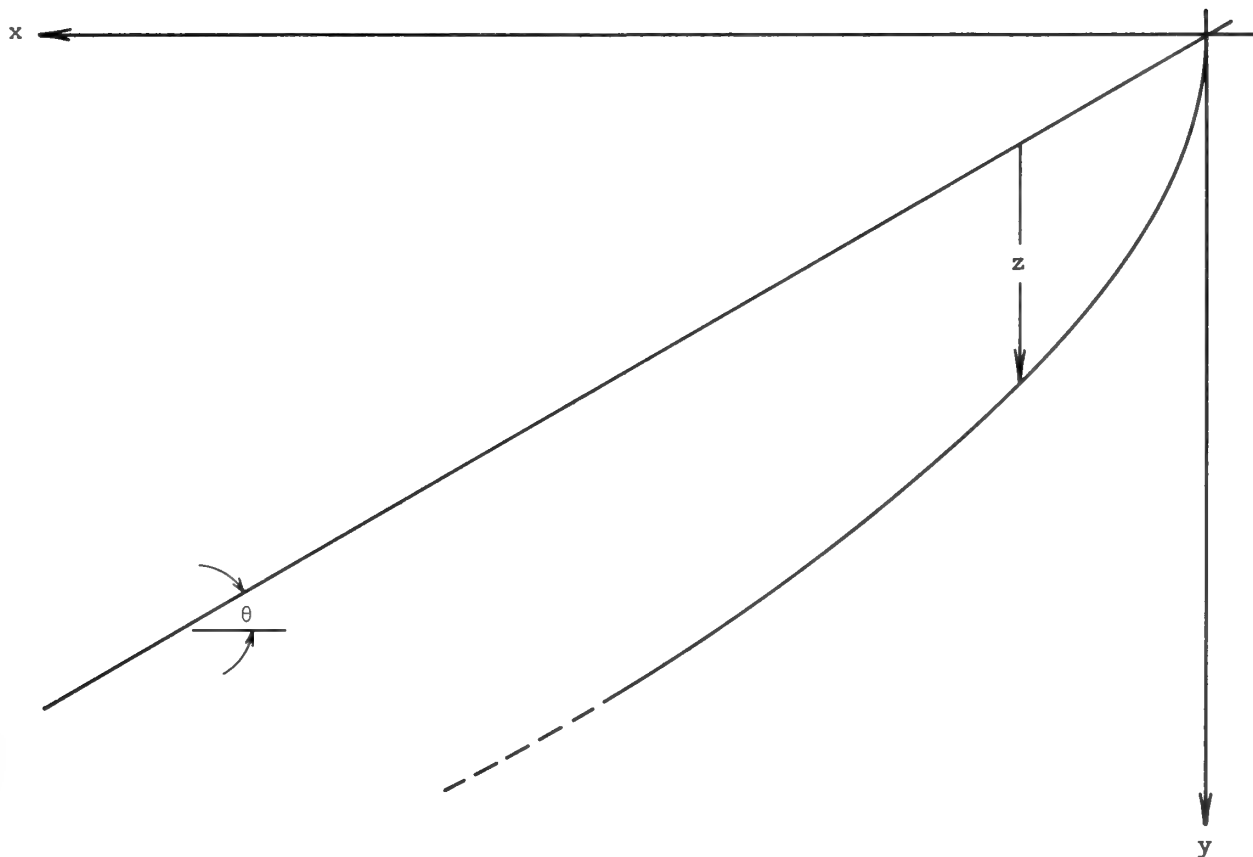


Figure 4.--Coordinate system (hypothetical failure surface).

It is convenient to express the equation for the failure surface in terms of rectangular coordinates. Considering the hypothetical failure surface shown in figure 4 and choosing the coordinates $x = y = 0$ to denote the intersection of the failure surface with the ground surface, the equation for the failure surface can be written

$$y = z + x \tan \theta \dots \dots \dots (21)$$

The slope of the failure surface is given by the expression

$$m = \frac{dy}{dx} \dots \dots \dots (22)$$

Taking the derivative of the equation for the failure surface, equation 21, the expression for x is obtained as follows:

$$m = \frac{dy}{dx} = \frac{dz}{dx} + \tan \theta \dots \dots \dots (23)$$

and

$$x = \int_0^x dx = \int_0^z \left(\frac{1}{m - \tan \theta} \right) dz \dots \dots \dots (24)$$

Combining equations 10, 15, 16, and 20, it can be shown that, for the general case where the point being considered lies below the phreatic surface,

$$\begin{aligned} \frac{1}{m - \tan \theta} &= \frac{-(\tan \theta + \tan \phi)}{(1 + \tan^2 \theta)} \\ &\cos^2 \theta \{ (z - z_w) [\gamma_b (\tan \theta + \tan \phi) \\ &\quad + \gamma_w \tan \theta] \\ &\quad + z_w \gamma_t (\tan \theta + \tan \phi) \\ &\quad + \frac{c (1 + \tan^2 \theta)^{\frac{1}{2}}}{\cos \phi \{ (z - z_w) [\gamma_b (\tan \phi - \tan \theta) \\ &\quad - \gamma_w \tan \theta] \\ &\quad + z_w \gamma_t (\tan \phi - \tan \theta) \\ &\quad + c (1 + \tan^2 \theta)^{\frac{1}{2}} \} } \dots \dots \dots (25) \end{aligned}$$

Similarly, for a point above the ground water level, or for the special case of no seepage,

$$\begin{aligned} \frac{1}{m - \tan \theta} &= \frac{-(\tan \theta + \tan \phi)}{(1 + \tan^2 \theta)} \\ &\cos^2 \theta \{ z \gamma_t (\tan \theta + \tan \phi) \\ &\quad + \frac{c (1 + \tan^2 \theta)^{\frac{1}{2}}}{\cos \phi \{ z \gamma_t (\tan \phi - \tan \theta) \\ &\quad + c (1 + \tan^2 \theta)^{\frac{1}{2}} \} } \dots \dots \dots (26) \end{aligned}$$

where, in each case, the (-) and (+) signs correspond to the passive and active situations, respectively.

Thus, from equations 24 and 25, for $z \geq z_w$,

$$\begin{aligned} x - x_0 &= A (\eta - \eta_0) \mp [B \frac{\sqrt{uv}}{b} \\ &\quad - \frac{Bk}{b \sqrt{-bb'}} \tan^{-1} \left(\frac{\sqrt{-bb' uv}}{bv} \right)]^n \dots \dots \dots (27) \\ &\quad \eta_0 \end{aligned}$$

where $\eta = z - z_w$, and η_0 and x_0 correspond to the depth $z = z_w$. Again, the (-) and (+) signs correspond to the passive and active cases, respectively, and the remaining parameters and variables are defined as follows:

$$A = \frac{-(\tan \theta + \tan \phi)}{(1 + \tan^2 \theta)} \dots \dots \dots (28a)$$

$$B = \cos^2 \theta (1 + \tan^2 \phi)^{\frac{1}{2}} \dots \dots \dots (28b)$$

$$a = z_w \gamma_t (\tan \phi - \tan \theta) + c (1 + \tan^2 \theta) \dots \dots \dots (29a)$$

$$b = \gamma_b (\tan \phi - \tan \theta) - \gamma_w \tan \theta \dots \dots \dots (29b)$$

$$a' = z_w \gamma_t (\tan \theta + \tan \phi) + c (1 + \tan^2 \theta) \dots \dots \dots (29c)$$

$$b' = \gamma_b (\tan \theta + \tan \phi) + \gamma_w \tan \theta \dots \dots \dots (29d)$$

$$u = a + b \eta \dots \dots \dots (29e)$$

$$v = a' + b' \eta \dots \dots \dots (29f)$$

$$k = a b' - a' b \dots \dots \dots (29g)$$

Likewise, from equations 24 and 26, for $z \leq z_w$ (or for the case of no seepage),

$$x - x_0 = A (z - z_0) + \left[B \frac{\sqrt{uv}}{b} - \frac{Bk}{b \sqrt{-bb'}} \tan^{-1} \left(\frac{\sqrt{-bb' uv}}{bv} \right) \right] \dots \dots \dots (30)$$

where x_0 and z_0 correspond to ground surface (i.e., $x_0 = z_0 = 0$) and the (-) and (+) signs correspond to the passive and active cases, respectively. Here A and B are as defined in equation 28, and the remaining parameters and variables are defined as follows:

$$a = c (1 + \tan^2 \theta) \dots \dots \dots (31a)$$

$$b = \gamma_t (\tan \phi - \tan \theta) \dots \dots \dots (31b)$$

$$a' = a = c (1 + \tan^2 \theta) \dots \dots \dots (31c)$$

$$b' = \gamma_t (\tan \theta + \tan \phi) \dots \dots \dots (31d)$$

$$u = a + b z \dots \dots \dots (31e)$$

$$v = a' + b' z \dots \dots \dots (31f)$$

$$k = a b' - a' b \dots \dots \dots (31g)$$

Equation 30 can be shown to agree with the solutions obtained using Frontard's approach.

Note that the above solutions are limited to situations where $c > 0$ and $\theta > \tan^{-1} \left[\frac{\gamma_b}{\gamma_{sat}} \tan \phi \right]$, where γ_{sat} is the saturated unit weight of soil below the water table ($\gamma_{sat} = \gamma_b + \gamma_w$). For the case of no seepage, the above solutions are limited to situations where $\theta > \phi$. For conditions other than these, special (and generally simpler) solutions can be found.

RESULTS

Representative failure curves obtained from equations 27 and 30 are shown in figure 5. For the particular slope and soil properties chosen for illustrative purposes in figure 5, the critical depth for the case of no seepage is 11.85 ft. The curves for two of the cases, $z_w = 5$ ft. and $z_w > 11.85$ ft., coincide to a depth of 5 ft.

The critical depths z_1 , z_2 , and z_3 are found from the expressions

$$z_1 = \frac{c}{\cos^2 \theta \{ \gamma_b (\tan \theta - \tan \phi) + \gamma_w \tan \theta \}} \quad \dots \quad (32)$$

$$z_2 = z_w + \frac{c - z_w \gamma_t \cos^2 \theta (\tan \theta - \tan \phi)}{\cos^2 \theta \{ \gamma_b (\tan \theta - \tan \phi) + \gamma_w \tan \theta \}} \quad \dots \quad (33)$$

$$z_3 = \frac{c}{\gamma_t \cos^2 \theta (\tan \theta - \tan \phi)} \quad \dots \quad (34)$$

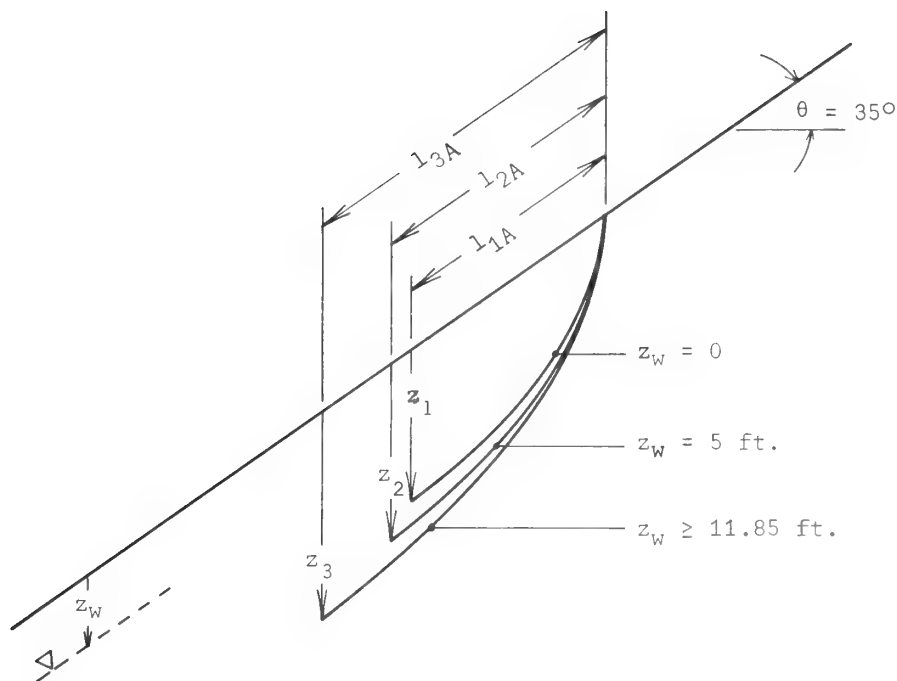
The values of l_{1A} , l_{1P} , l_{2A} , l_{2P} , l_{3A} , and l_{3P} are found by dividing the appropriate horizontal lengths, x , by $\cos \theta$.

The ratios l_{3A}/z_3 , and l_{3P}/z_3 are plotted in figures 6a and 6b to illustrate their dependency upon ϕ and θ . For cases of no seepage, the length-to-depth ratios are independent of the soil properties, c and γ_t . Auxiliary curves have been plotted in figures 6a and 6b to illustrate the dependence of length-to-depth ratios on the values of $(\theta - \phi)$. As $(\theta - \phi)$ approaches zero, the length-to-depth ratios increase toward infinity.

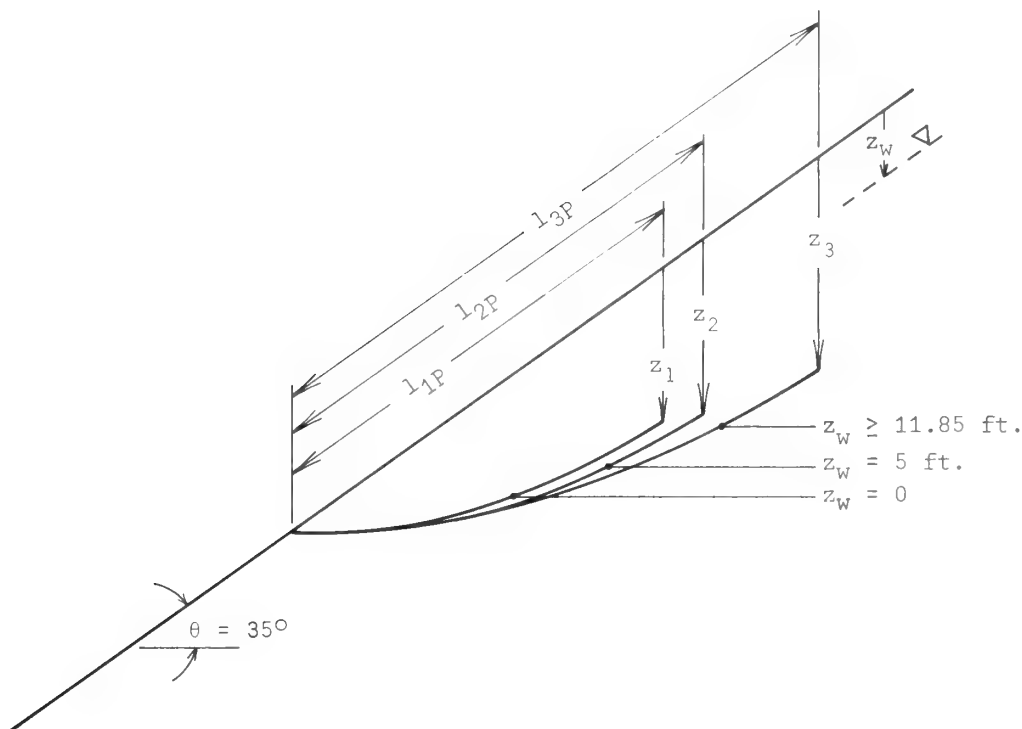
The length-to-depth ratios depend upon the buoyant unit weight of the soil, γ_b , as well as upon ϕ and θ for cases where seepage occurs throughout the soil profile and flow is parallel to ground slope. The relationships of l_{1A}/z_1 and l_{1P}/z_1 to ϕ and θ for buoyant unit weights of 30, 60, and 90 p.c.f. are shown in figures 7, 8, and 9, respectively. As in the case of no seepage, these length-to-depth ratios are independent of soil cohesive strength, c .

Although not illustrated in figures 7, 8, and 9, the length-to-depth ratios increase rapidly toward infinity as the values of $\tan \theta - \frac{\gamma_b}{\gamma_{sat}} \tan \phi$ approach zero.

Finally, it is of interest to examine the variations of length-to-depth ratios as z_w varies between 0 and z_3 . Figure 10 shows the relationships of l_{2A}/z_2 , l_{2P}/z_2 , and $(l_{2A} + l_{2P})/z_2$ to z_w for the particular slope and soil conditions assumed in figure 5. The actual variations of lengths l_{2A} , l_{2P} , and $(l_{2A} + l_{2P})$ with z_w are also shown in figure 10.



(a) Active curves



(b) Passive curves

Figure 5.--Failure curves for infinite slope with water table at various depths.
 (Assumed soil properties: $c = 500 \text{ p.s.f.}$, $\phi = 10^\circ$, $\gamma_t = 120 \text{ p.c.f.}$, and $\gamma_b = 80 \text{ p.c.f.}$ Critical depths: $z_1 = 8.70 \text{ ft.}$, $z_2 = 10.03 \text{ ft.}$, $z_3 = 11.85 \text{ ft.}$).

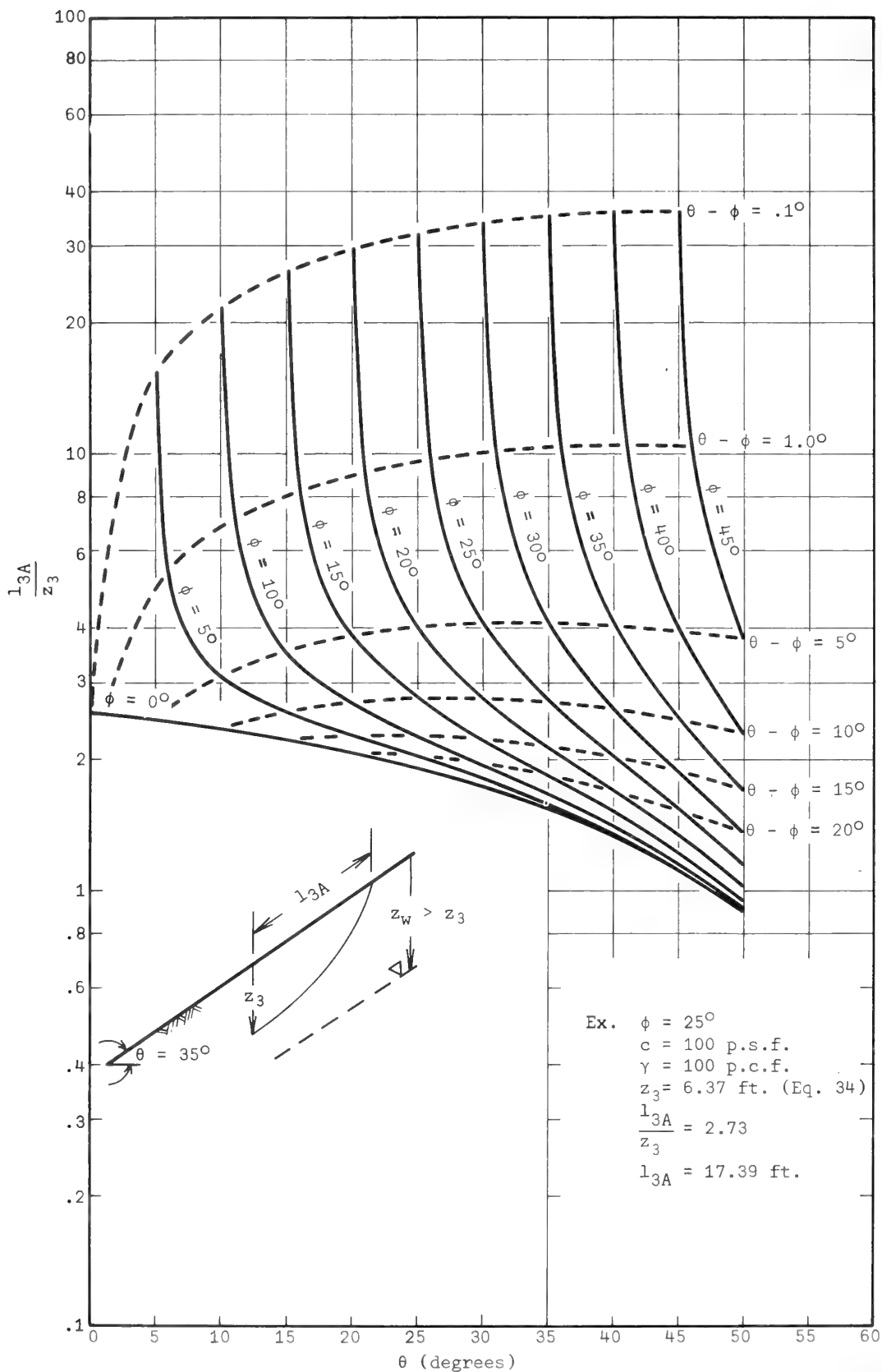


Figure 6a.--Length-to-depth ratios for cases of no seepage, active curves (i.e., $z_w >$ "dry" critical depth).

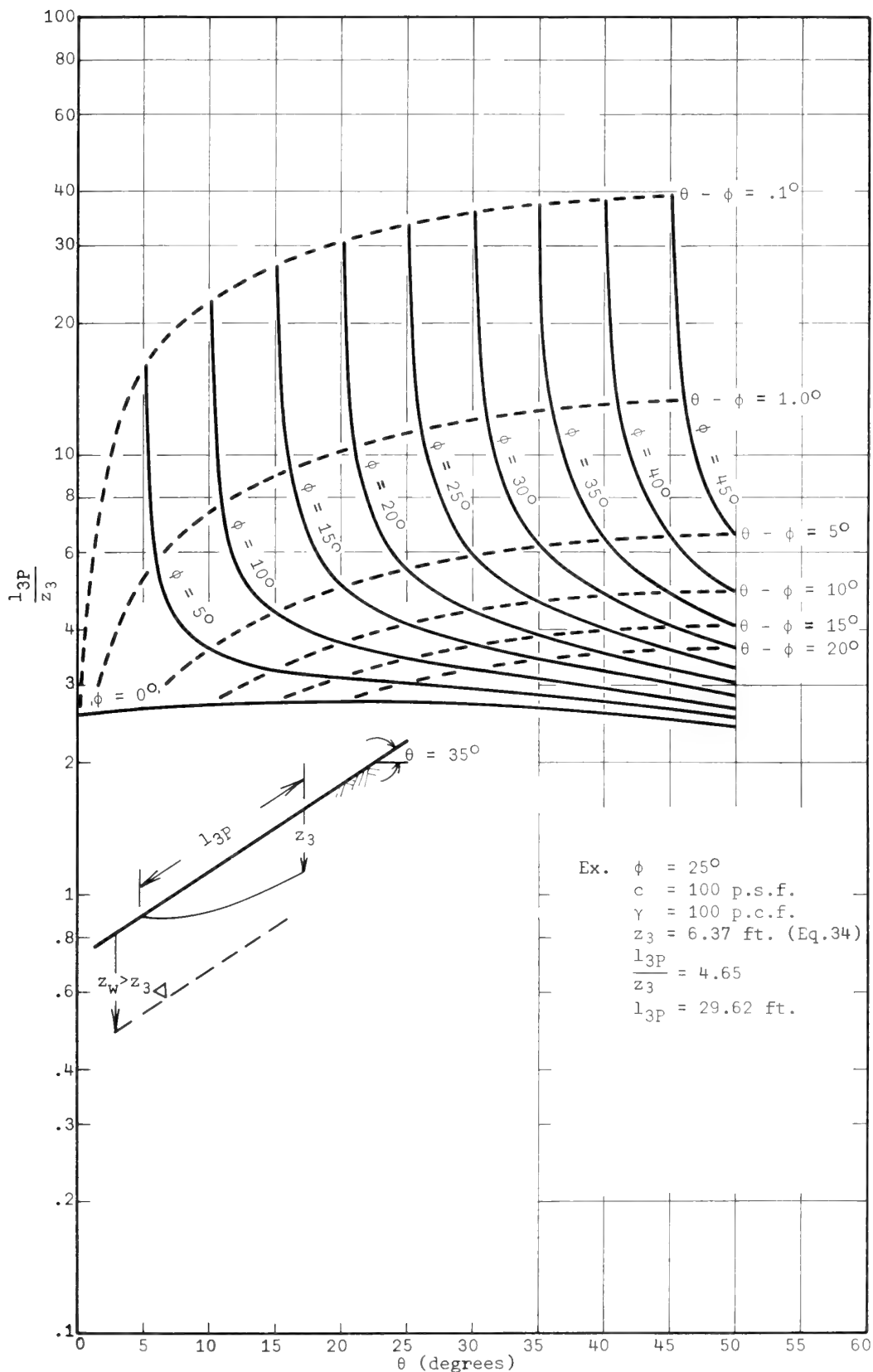


Figure 6b.--Length-to-depth ratios for cases of no seepage, passive curves (i.e., $z_w >$ "dry" critical depth).

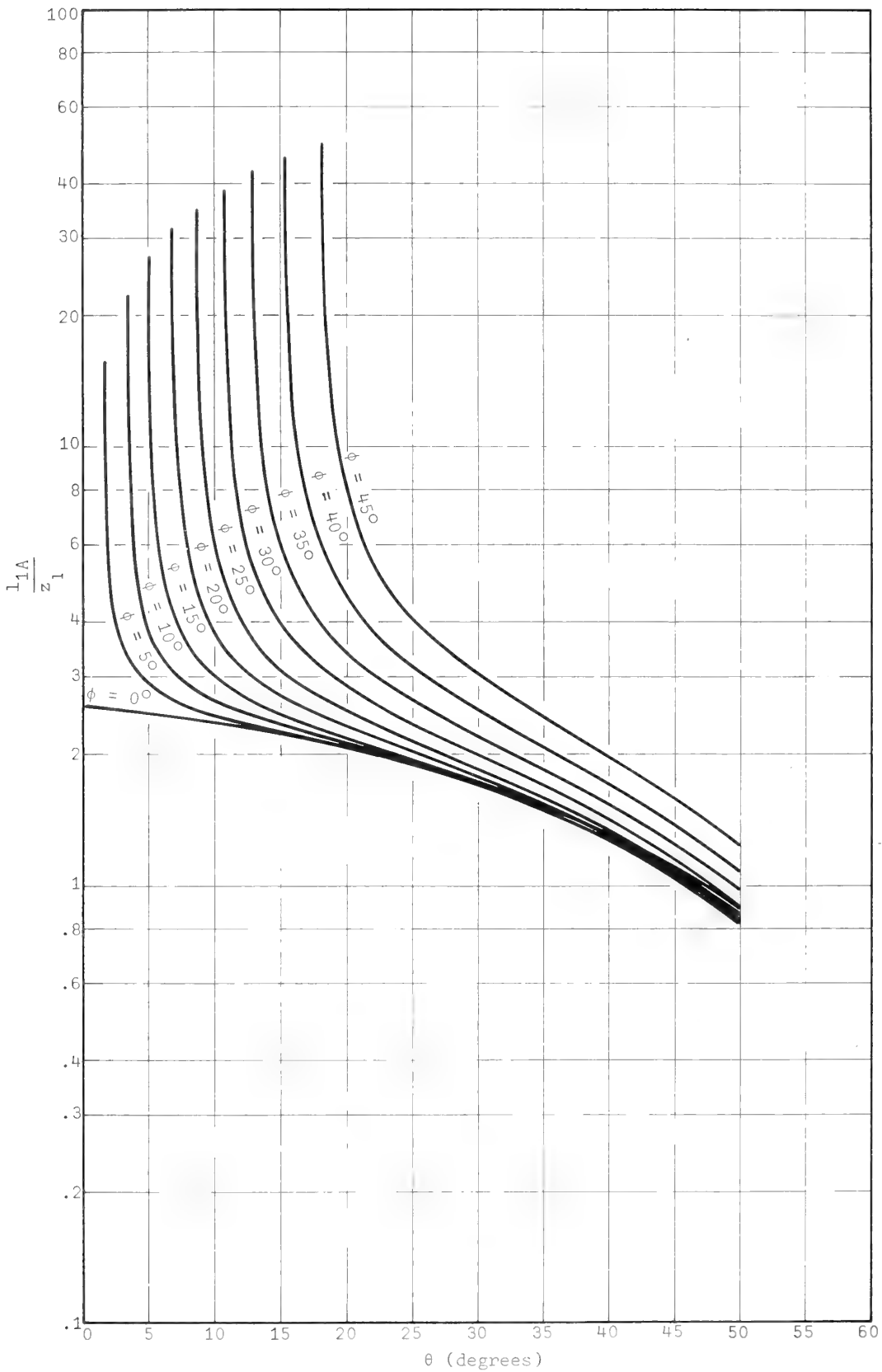


Figure 7a.--Length-to-depth ratios for $z_w = 0$, $\gamma_b = 30$ p.c.f., active curves.

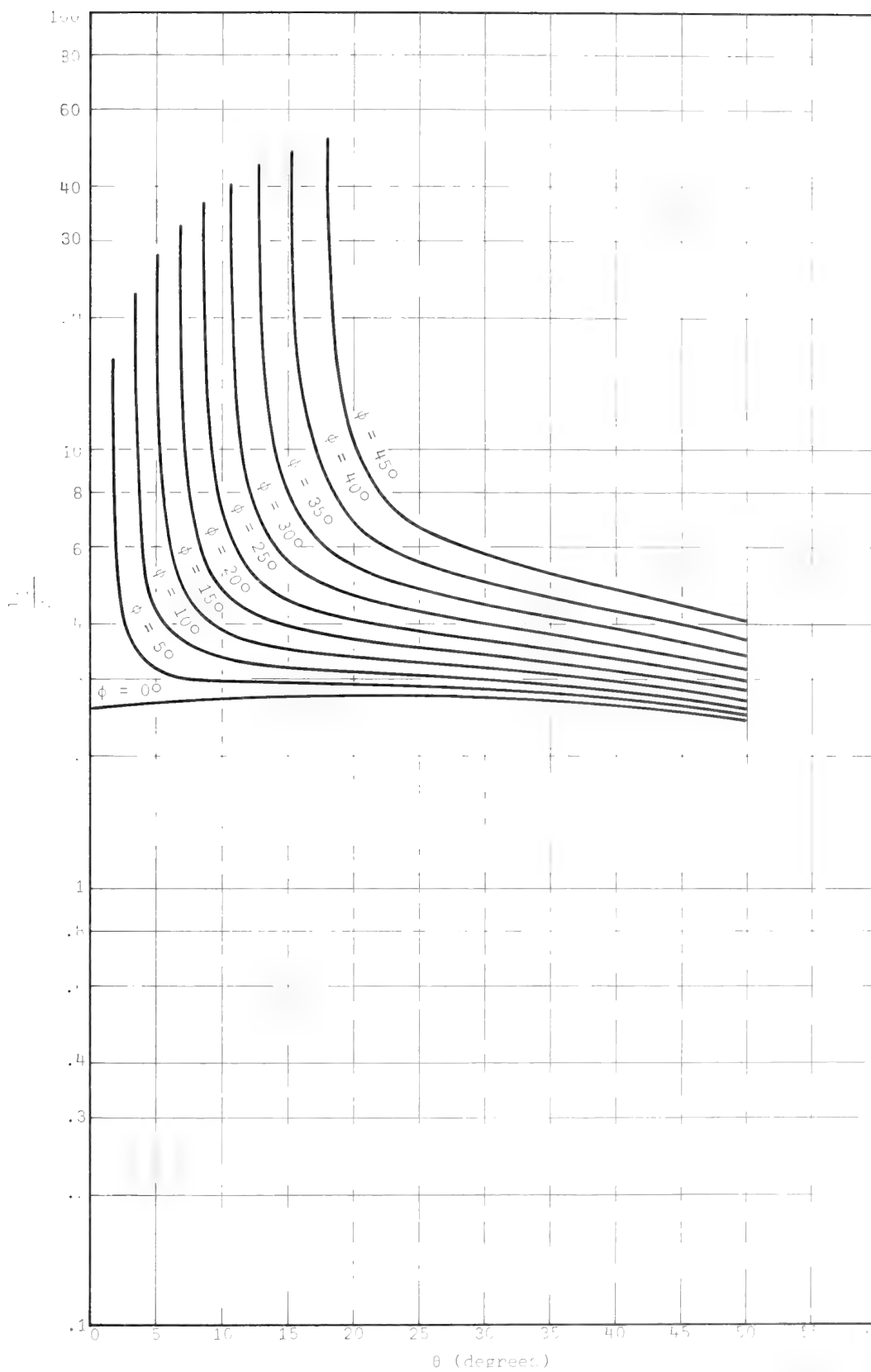


Figure 7b.--Length-to-depth ratios for $z_w = 0$, $\gamma_b = 30$ r.c.f., passive curves.

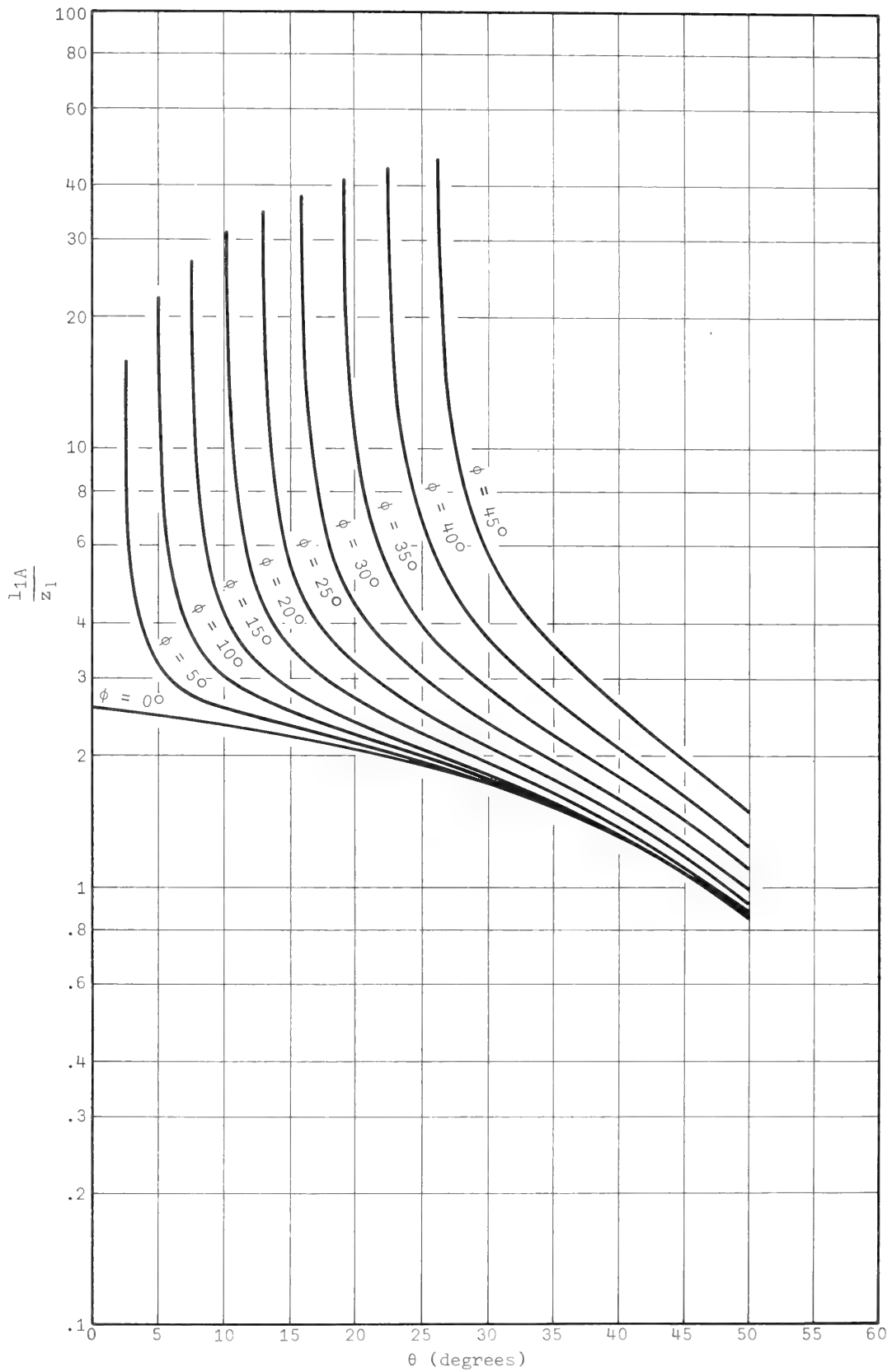


Figure 8a.--Length-to-depth ratios for $z_w = 0$, $\gamma_b = 60$ p.c.f., active curves.

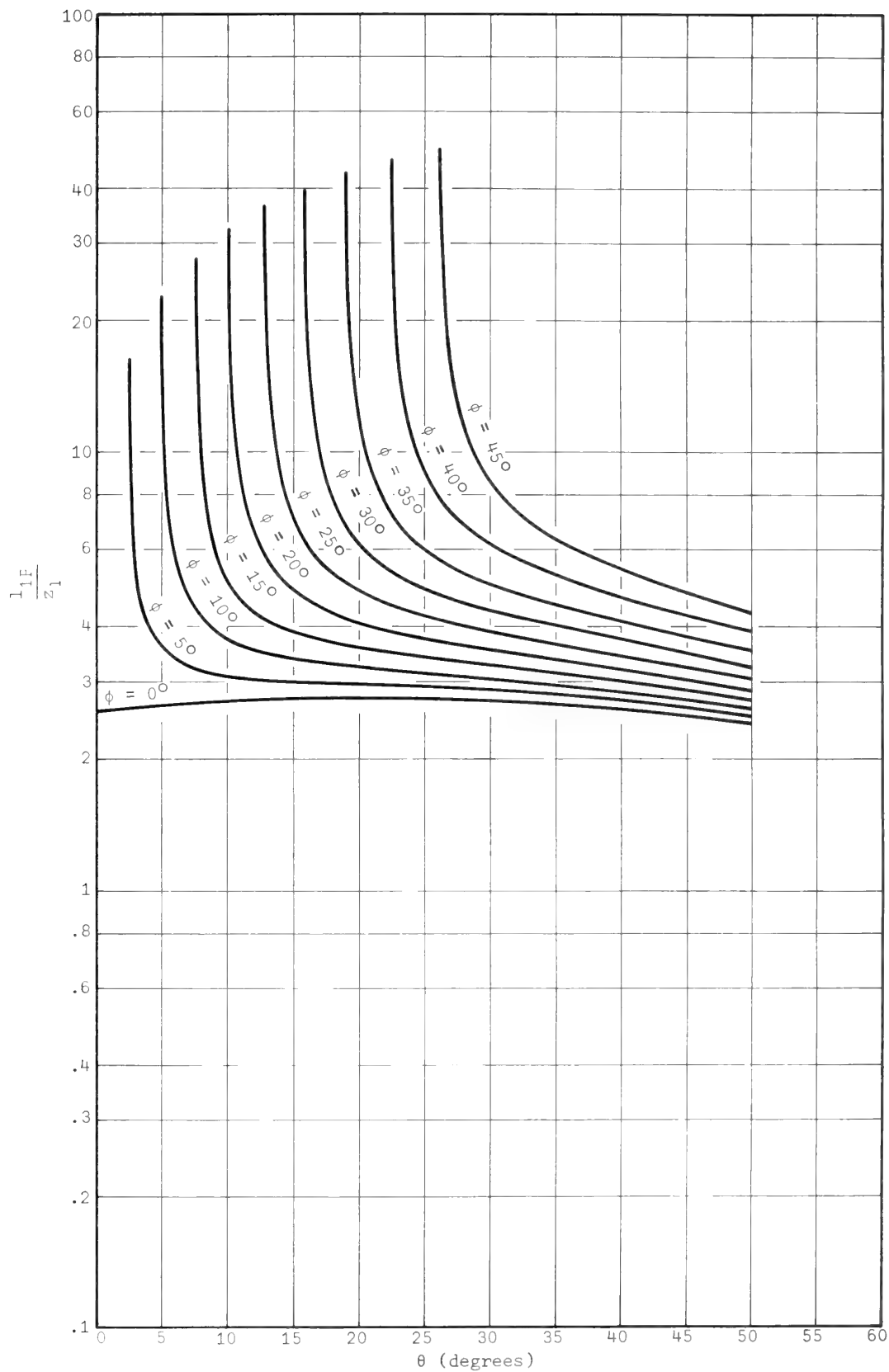


Figure 8b.--Length-to-depth ratios for $z_w = 0$, $\gamma_b = 60$ p.c.f., passive curves.

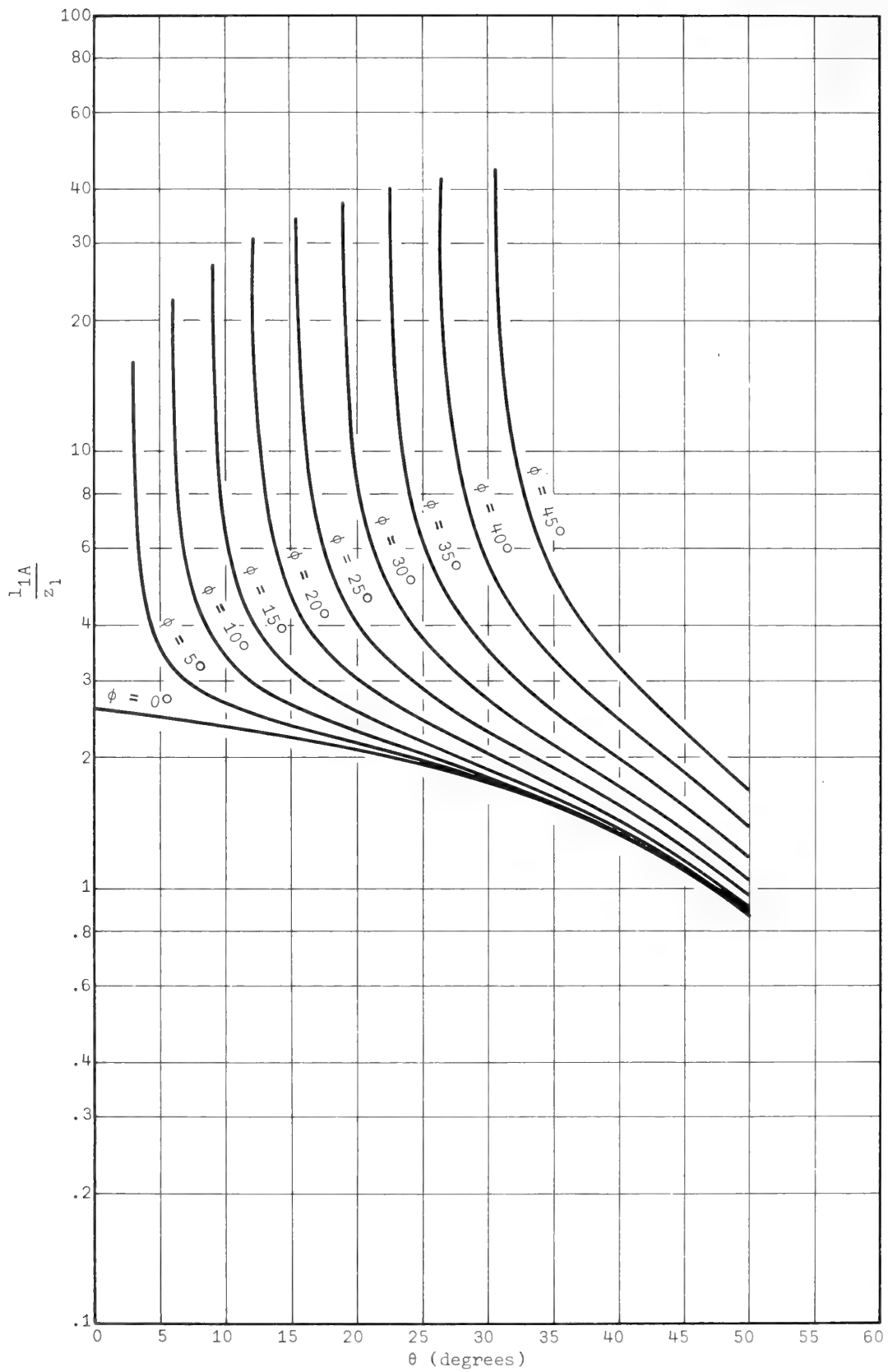


Figure 9a.--Length-to-depth ratios for $z_w = 0$, $\gamma_b = 90$ p.c.f., active curves.

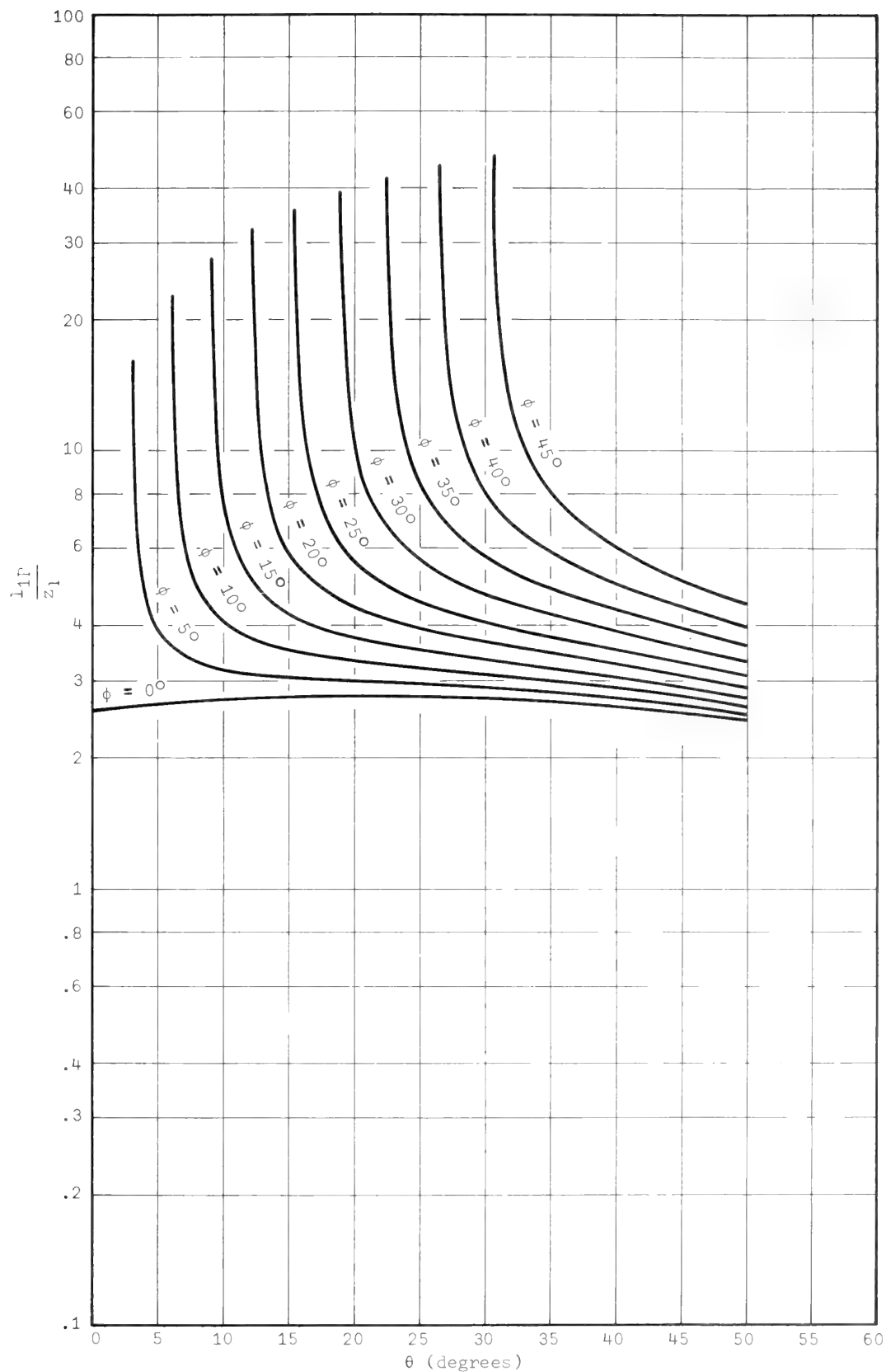


Figure 9b.--Length-to-depth ratios for $z_w = 0$, $\gamma_b = 90$ p.c.f., passive curves.

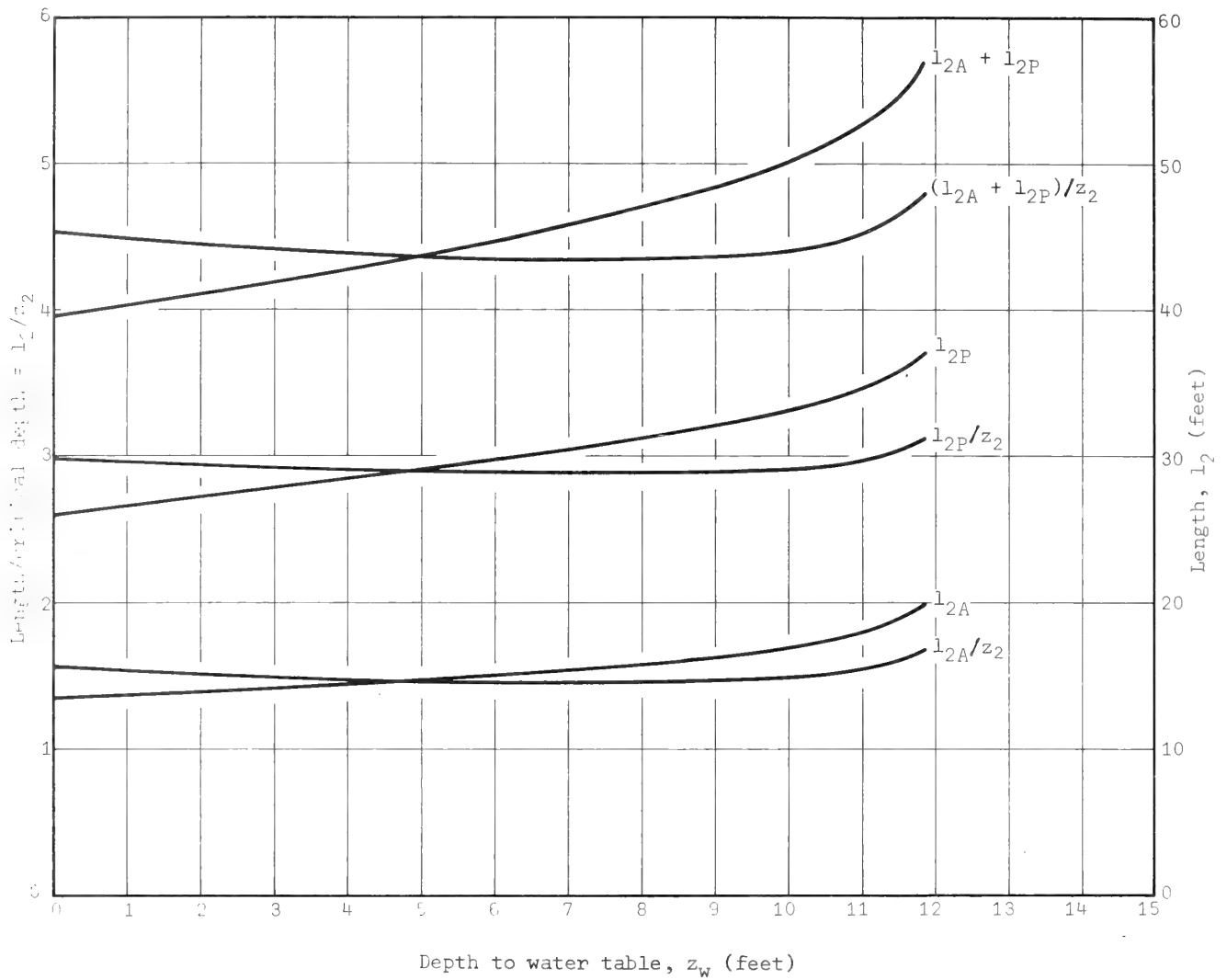


Figure 10.--Lengths and length-to-depth ratios versus z_w , for slope and soil conditions assumed in figure 5.

DISCUSSION

The results presented in figures 6a and 6b indicate that the combined active and passive length-to-critical depth ratios for cases of no seepage are less than 25 when $\theta - \phi > 1^\circ$ and $\phi < 45^\circ$. Figures 7, 8, and 9 illustrate that seepage reduces the length-to-depth ratios. Figures 5 and 10 illustrate that seepage reduces both the lengths of the failure surfaces and the critical depths. However, the minimum length-to-depth ratios can occur at intermediate seepage conditions (i.e., when $0 < z_w < z_{cr}$).

Although the shapes of the derived failure surfaces appear similar to those observed in real circumstances (e.g., see fig. 5), these active and passive failure surfaces cannot simply be combined, as was correctly pointed out by Terzaghi. If, as stated by Taylor, the active and passive states must exist simultaneously at the upper and lower ends, respectively, of a real failure, then the infinite slope theory remains an unsatisfactory means for predicting the state of stress within any real slopes in which failure is impending.

Consider the stresses on planes normal to the slope. Figure 11 illustrates these stresses for both the active and passive conditions, for the soil and slope conditions assumed in figure 5, without seepage. Note that the active and passive stress states are identical at the critical depth. This represents the depth where $\bar{\sigma}_a$ and τ_a are on the strength envelope (see fig. 2); hence, only one circle of stress is possible on the Mohr diagram. Note also that these stress distributions are not linear, as was assumed by Terzaghi.

Because of moment equilibrium, the shear stresses on planes normal to the slope are identical for both the active and passive conditions as well as for all intermediate stress conditions; only the normal stresses differ (except at the critical depth). Letting n and s represent the coordinates normal and parallel to the slope, respectively, (fig. 11) then from equation 11,

$$\begin{aligned} \bar{\sigma}_s &= \bar{\sigma}_n (1 + 2 \tan^2 \phi) + 2 c \tan \phi \\ &\pm \frac{2}{\cos \phi} \{ \bar{\sigma}_n^2 (\tan^2 \phi - \tan^2 \theta) \\ &\quad + 2 \bar{\sigma}_n (c \tan \phi - S \tan \theta) + (c^2 - S^2) \}^{\frac{1}{2}} \dots (35) \end{aligned}$$

where the (+) and (-) signs are associated with the passive and active states, respectively.

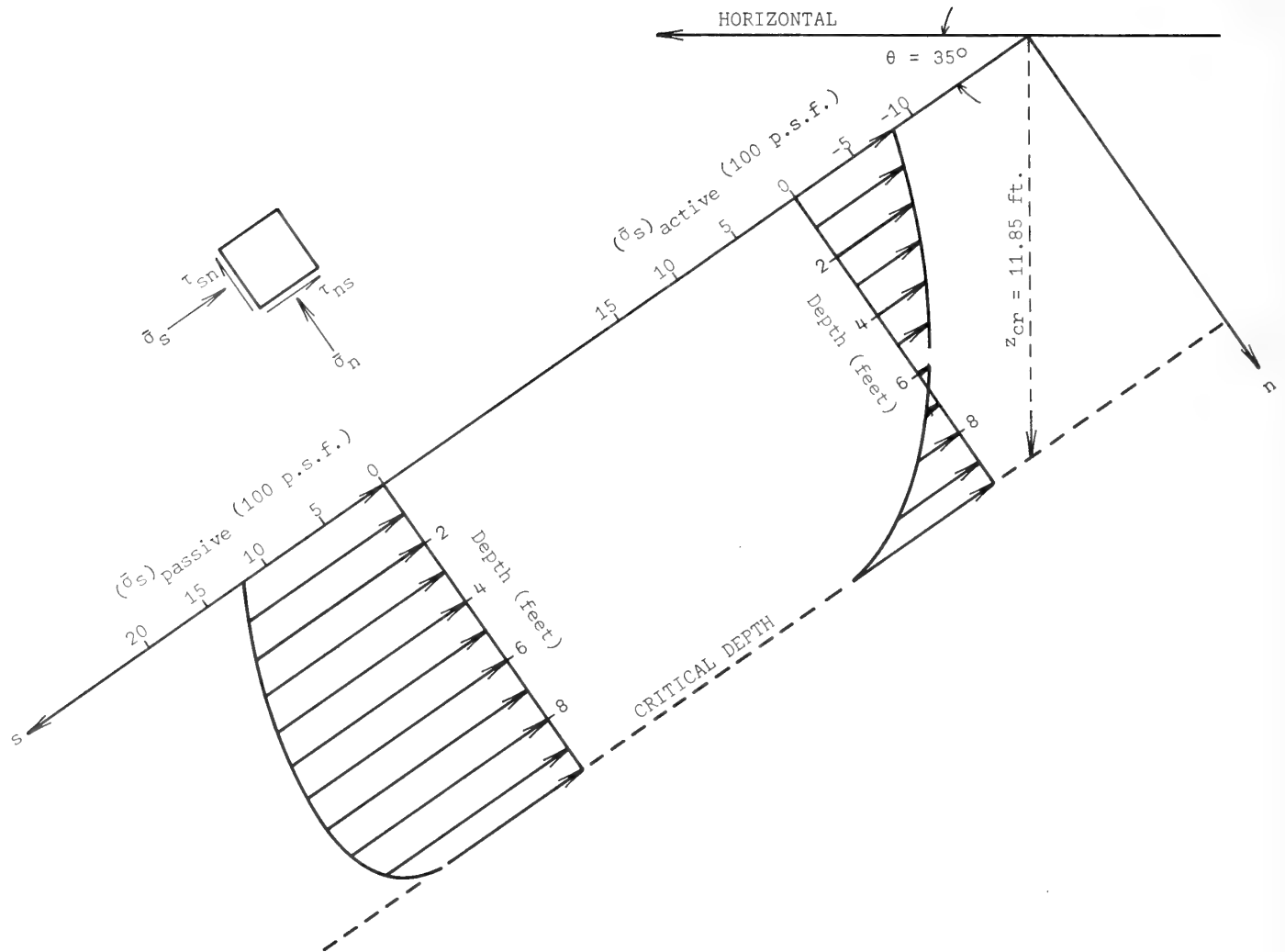


Figure 11.--Distribution of active and passive stresses on planes normal to ground slope under assumptions of the infinite slope theory ($\theta = 35^\circ$, $\phi = 10^\circ$, $c = 500$ p.s.f., $\gamma_t = 120$ p.c.f., $z_w > 11.85$ ft.).

It will be recalled from equation 10 that, in the case of seepage, S is the seepage force per unit area. However, it is convenient to consider S as simply an incremental shear stress which may be expressed

$$S = \tau_{ns} - \bar{\sigma}_n \tan \theta \quad \dots \dots \dots (36)$$

In cases without seepage, and under the assumptions of the infinite slope theory, $\tau_{ns} = \bar{\sigma}_n \tan \theta$, and therefore $S = 0$.

Combining equations 35 and 36,

$$\begin{aligned} \bar{\sigma}_s = & \bar{\sigma}_n (1 + 2 \tan^2 \phi) + 2 c \tan \phi \\ & \pm \frac{2}{\cos \phi} \{ (\bar{\sigma}_n \tan \phi + c)^2 - \tau_{ns}^2 \}^{\frac{1}{2}} \quad \dots \dots \dots (37) \end{aligned}$$

where, again, the (+) and (-) signs are associated with the passive and active conditions, respectively.

From equilibrium considerations,

$$\frac{\partial \tau_{ns}}{\partial n} = \gamma \sin \theta - \frac{\partial \bar{\sigma}_s}{\partial s} \quad \dots \dots \dots (38a)$$

$$\frac{\partial \bar{\sigma}_n}{\partial n} = \gamma \cos \theta - \frac{\partial \tau_{sn}}{\partial s} \quad \dots \dots \dots (38b)$$

$$\tau_{ns} = \tau_{sn} \quad \dots \dots \dots (38c)$$

Under the assumptions of the infinite slope theory, $\frac{\partial \bar{\sigma}_s}{\partial s} = \frac{\partial \tau_{sn}}{\partial s} = 0$, and the stresses within the slope are given simply as

$$\tau_{ns} = \tau_{sn} = \gamma n \sin \theta \quad \dots \dots \dots (39a)$$

$$\bar{\sigma}_n = \gamma n \cos \theta \quad \dots \dots \dots (39b)$$

$$\begin{aligned} & \bar{\sigma}_n (1 + 2 \tan^2 \phi) + 2 c \tan \phi \\ & - \frac{2}{\cos \phi} \{ (\bar{\sigma}_n \tan \phi + c)^2 - \tau_{ns}^2 \}^{\frac{1}{2}} \leq \bar{\sigma}_s \\ & \leq \bar{\sigma}_n (1 + 2 \tan^2 \phi) + 2 c \tan \phi \\ & + \frac{2}{\cos \phi} \{ (\bar{\sigma}_n \tan \phi + c)^2 - \tau_{ns}^2 \}^{\frac{1}{2}} \quad \dots \dots \dots (39c) \end{aligned}$$

where $\bar{\sigma}_s$ remains constant at all locations in the slope at a given depth.

However, if it is assumed that $\bar{\sigma}_s$ increases uniformly from the active to the passive condition in length of slope, L , then

$$\begin{aligned} \frac{\partial \bar{\sigma}_s}{\partial s} &= \frac{(\bar{\sigma}_s)_{\text{passive}} - (\bar{\sigma}_s)_{\text{active}}}{L} = \\ &= \frac{4}{L \cos \phi} \{ (\bar{\sigma}_n \tan \phi + c)^2 - \tau_{ns}^2 \}^{\frac{1}{2}} \quad \dots \dots \dots (40) \end{aligned}$$

whereby from equation 38a,

$$\frac{\partial \tau_{ns}}{\partial n} = \gamma \sin \theta - \frac{4}{L \cos \phi} \{ (\bar{\sigma}_n \tan \phi + c)^2 - \tau_{ns}^2 \}^{\frac{1}{2}} \quad \dots \dots \dots (41)$$

Assuming $\frac{\partial \tau_{sn}}{\partial s} = 0$, as in the infinite slope theory, then $\bar{\sigma}_n = \gamma_n \cos \theta$, and equation 41 becomes

$$\frac{d \tau_{ns}}{d n} = \gamma \sin \theta - \frac{4}{L \cos \phi} \{(\gamma_n \cos \theta \tan \phi + c)^2 - \tau_{ns}^2\}^{\frac{1}{2}} \dots (42)$$

This provides a rational, though admittedly oversimplified and slightly inaccurate, means for allowing the active and passive states of stress to occur simultaneously at the top and bottom ends, respectively, of a failure mass in a long slope. It is simplified in that the shear stresses on planes parallel and normal to the slope are constant for a given depth, as in the infinite slope theory; and in that the normal stresses on planes normal to the slope increase at a constant rate from the active to the passive values. It is inaccurate in that it does not enable development of the failure surfaces connecting the critical depth with ground surface at the ends of the failure mass. However, this inaccuracy should not be too serious if L is large relative to the lengths required to develop the connecting failure surfaces.

The effect of this modification of the infinite slope theory is to increase the critical depth. Unfortunately, attempts to solve equation 42 for τ_{ns} have been unsuccessful. However, a finite difference approximation has been programmed to enable determination of the τ_{ns} versus depth relationship as well as the critical depth for any particular slope and soil.

Figure 12 illustrates the effects of varying L on the τ_{ns} vs. $\bar{\sigma}_n$ relationship and on the critical depth for the slope and soil conditions assumed in figure 5. No general conclusions can be drawn from figure 12 because it represents a particular case. However, the reader can easily conduct a similar analysis for any other slope and soil by programming a finite difference solution to the above differential equation. It will merely be noted that, for the slope and soil conditions assumed in figures 5 and 12, transition lengths less than about 100 times the critical depth of 11.85 ft. result in rapidly increasing modified critical depths.

It will be recalled from figure 5 that the critical depth is reduced from 11.85 to 8.7 ft., or about 25 percent, when $z_w = 0$, or seepage is occurring throughout the soil profile. However, according to figure 12, if the transition length was about 170 ft. (or approximately 14 times the conventional critical depth of 11.85 ft.), then the modified critical depth would be about 25 percent greater than the conventional critical depth. Thus, a depth of 11.85 ft. on a slope of this length (170 ft.) might be safe under a condition of seepage throughout the soil profile even though the conventional theory would predict an unsafe condition.

Discussion thus far has been limited to uniform, uninterrupted slopes. Usually, the major interest in natural slopes is with regard to their response to excavations, or cuts, and to such other activities as timber harvesting. Recent advances in the application of the finite element method of analysis seem to hold the greatest promise for assessing the response of natural slopes to such alterations. Nevertheless, it still remains necessary to be able to describe the state of stress within the slope before modification as well as the *in situ* material properties. Having done so, it may be possible to use the simple transition-length approach to assess the stability of many cut slopes.

Accuracy of any method of analysis remains limited by the ability to measure or predict stress history and soil characteristics. Further, more knowledge about seepage and pore pressures is needed, especially in the realm of unsaturated flow. Also, the mechanisms of creep and progressive failure require considerably more study and elucidation. It is apparent that any accurate, rational method of analysis will ultimately have to combine rheological and soil moisture characteristics with effective strength characteristics in order to fully account for the behavior of natural slopes.

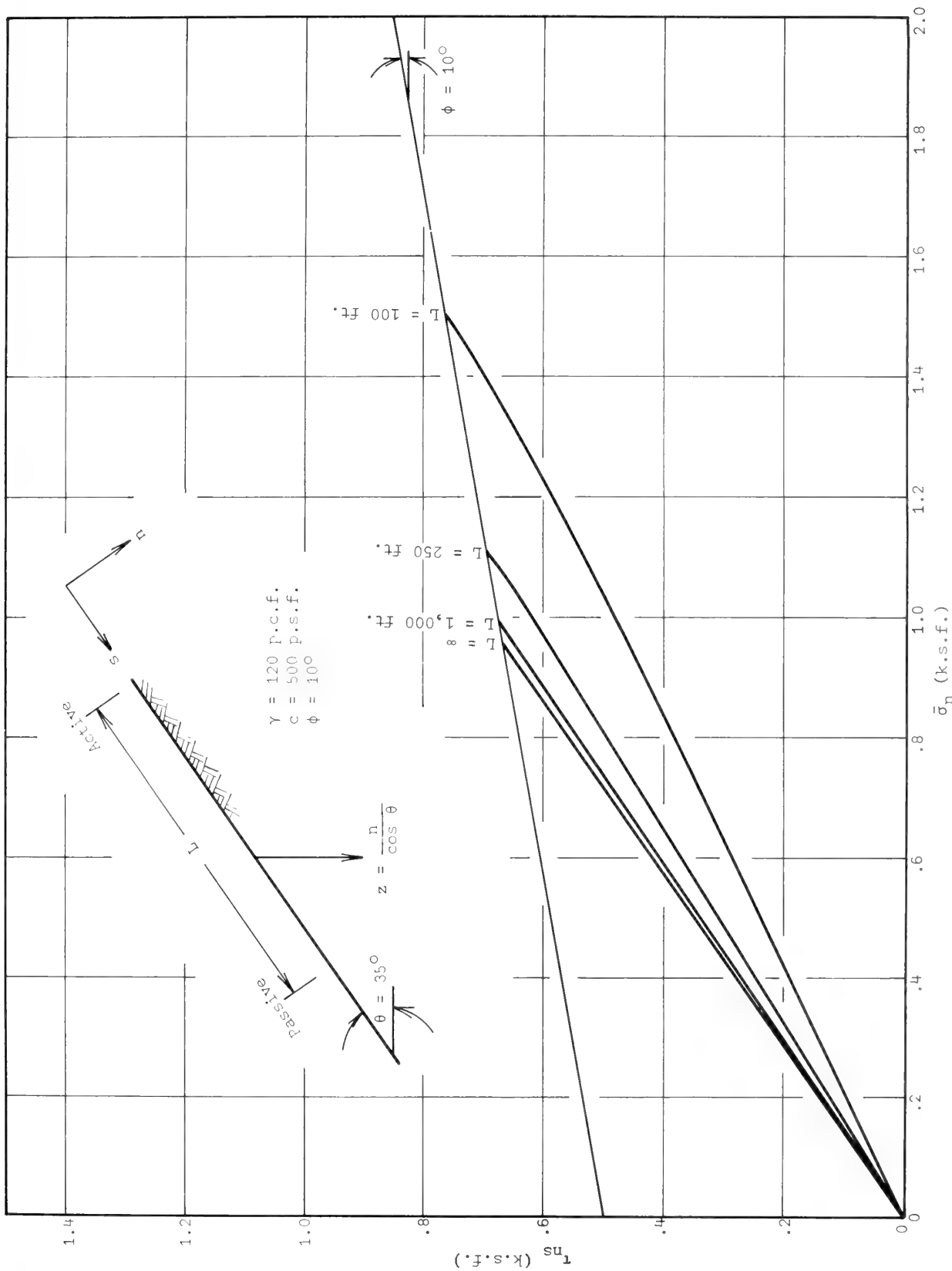


Figure 12a.--Effect of varying transition length, L , on τ_{ns} vs. σ_n relationship for slope and soil conditions assumed in figure 11.

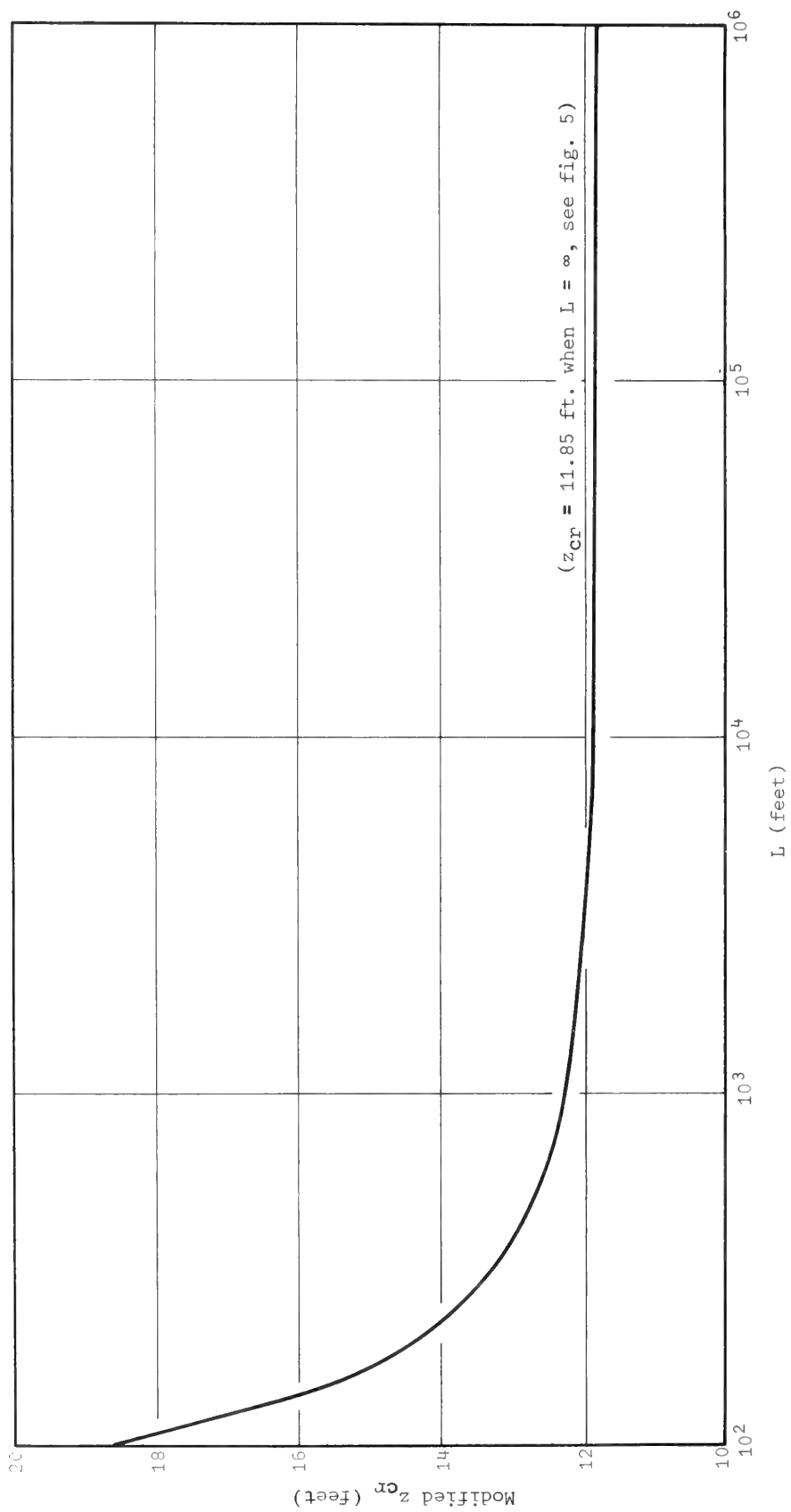


Figure 12b.--Critical depth, z_{cr} , vs. transition length, L , for slope and soil conditions assumed in figure 11.

CONCLUSIONS

Mathematical relationships have been derived to describe the failure surfaces in slopes where the assumptions of the infinite slope theory are valid. These relationships are general in that they can be applied to conditions of seepage wherein the piezometric level is at any depth below ground surface, provided that seepage is parallel to the slope. The relationships can also be adapted to layered systems, with or without seepage, provided that layering is parallel to the slope. This can most easily be accomplished by treating the material lying above a particular layer as simply a surcharge, or by converting the overlying layers to an equivalent depth of the material for which computations are being made. Use of a digital computer will facilitate computations.

Applicability of the infinite slope theory to real slopes has been discussed briefly, and it is concluded that no general "rule of thumb" or other criteria can be used to assess its applicability. Each slope must be judged according to its length and according to the soil characteristics and seepage conditions.

A simplified procedure to account for transition from the active to passive stress states within a long failure mass also has been presented and discussed. It is suggested that the simple transition-length idea applied herein to uninterrupted slopes might also be useful in the analysis of some cut slopes.

General solutions for the failure surfaces in slopes where the assumptions of the infinite slope theory are valid have been derived. The solutions are applicable to layered systems and to any seepage conditions provided that both layering and ground water flow are parallel to ground slope. It is concluded that the infinite slope theory is of limited applicability unless modified. A modification to enable transition from the active to passive stress states is suggested.

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APPENDIX

Symbols Used in This Report

A, a, a' = parameters in equations for failure surfaces;

B, b, b' = parameters in equations for failure surfaces;

c = cohesive strength;

k = parameter in equations for failure surfaces;

L = length of transition from active to passive stress states, measured parallel to ground slope;

l_{1A} = length of active failure surface measured parallel to ground slope for case when phreatic surface coincides with ground surface;

l_{1P} = length of passive failure surface measured parallel to ground slope for case when phreatic surface coincides with ground surface;

l_{2A} = length of active failure surface measured parallel to ground slope for case when phreatic surface lies between ground surface and critical depth;

l_{2P} = length of passive failure surface measured parallel to ground slope for case when phreatic surface lies between ground surface and critical depth;

l_{3A} = length of active failure surface measured parallel to ground slope for case when phreatic surface lies below critical depth;

l_{3P} = length of passive failure surface measured parallel to ground slope for case when phreatic surface lies below critical depth;

$m = \tan \alpha_f$;
 n = coordinate normal to ground surface;
 \bar{n} = normal stress acting on rupture plane;
 p = resultant stress acting on plane parallel to ground slope;
 r = resultant stress acting on vertical plane;
 S = seepage force per unit area, and generalized incremental shear stress;
 s = coordinate parallel to ground surface;
 t = tangential stress acting on rupture plane;
 u, v = parameters in equations for failure surfaces;
 x = horizontal coordinate of point on failure surface measured from intersection of failure surface with ground surface;
 x_0 = horizontal coordinate of intersection of failure surface with phreatic surface in (x, y) coordinate system;
 x' = coordinate of point on failure surface measured parallel to ground slope from intersection of failure surface with ground surface;
 y = vertical coordinate of point on failure surface measured from intersection of failure surface with ground surface;
 y' = coordinate of point on failure surface measured vertically below ground surface;
 z = vertical depth below ground surface;
 z_1 = critical depth when phreatic surface coincides with ground surface;
 z_2 = critical depth when phreatic surface lies between ground surface and critical depth;
 z_3 = critical depth when phreatic surface lies below critical depth;
 z_{cr} = critical depth;
 z_w = vertical depth of phreatic surface below ground surface;
 α = slope of line from origin of planes to point on Mohr stress circle with respect to horizontal;
 α' = angle between tangent to rupture surface and ground surface;
 α_f = angle of line connecting origin of planes with point of tangency of Mohr stress circle with strength envelope;
 β = angle of obliquity beneath phreatic surface;
 γ_b = buoyant unit weight of soil below phreatic surface;

γ_{sat} = saturated unit weight of soil beneath phreatic surface;
 γ_t = unit weight of soil above phreatic surface;
 γ_w = unit weight of water;
 η = transformed coordinate ($= z - z_w$);
 η_0 = transformed coordinate at phreatic surface ($= 0$);
 θ = slope of ground surface;
 λ, λ_0 = parameters in Frontard's solution;
 $\bar{\sigma}$ = effective normal stress;
 $\bar{\sigma}_0$ = normal stress coordinate of center of Mohr stress circle;
 $\bar{\sigma}_a$ = effective normal stress on plane parallel to ground surface;
 $\bar{\sigma}_b$ = effective normal stress coordinate of origin of planes on Mohr stress circle;
 $\bar{\sigma}_c$ = effective normal stress on plane oriented at angle α and depth z ;
 $\bar{\sigma}_n$ = effective normal stress on plane parallel to ground surface;
 $\bar{\sigma}_s$ = effective normal stress on plane normal to ground surface;
 $\bar{\sigma}_x$ = effective normal stress coordinate of point of tangency of Mohr stress circle with strength envelope;
 τ = shear stress;
 τ_0 = shear stress coordinate of center of Mohr stress circle ($= 0$);
 τ_a = shear stress on plane parallel to ground surface;
 τ_b = shear stress coordinate of origin of planes on Mohr stress circle;
 τ_c = shear stress on plane oriented at angle α and depth z ;
 τ_{ns} = shear stress on plane parallel to ground surface;
 τ_{sn} = shear stress on plane normal to ground surface;
 τ_x = shear stress coordinate of point of tangency of Mohr stress circle with strength envelope; and
 ϕ = angle of internal friction.

GONSIOR, MICHAEL J., WILLIAM S. HARTSOG, and GLEN L. MARTIN
1974. Failure surfaces in infinite slopes. USDA For. Serv. Res.
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General solutions for the failure surfaces in slopes where the assumptions of the infinite slope theory are valid have been derived. The solutions are applicable to layered systems and to any seepage conditions provided that both layering and ground water flow are parallel to ground slope. It is concluded that the infinite slope theory is of limited applicability unless modified. A modification to enable transition from the active to passive stress states is suggested.

OXFORD: 38:383.6. KEYWORDS: Slope stability, failure surfaces, infinite slope theory.

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